A class of mixed finite element methods based on the Helmholtz decomposition

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The construction of non-conforming finite element methods (FEMs) is motivated by robust discretizations and mass conservation properties in the simulation of solid and fluid mechanics and by low-order ansatz spaces for higher-order problems as the biharmonic problem for the Kirchhoff plate in structural mechanics. A natural generalization to higher polynomial degrees which preserves the inherent properties of the discretizations is not known so far.

This talk generalizes the non-conforming FEMs of Morley and Crouzeix and Raviart by novel mixed formulations for mth-Laplace equations of the form $(-1)^m \Delta^m u = f$ for arbitrary $m = 1, 2, 3, \ldots$ These formulations are based on a new Helmholtz decomposition which decomposes an unstructured tensor field into a higher-order derivative and a curl. The new formulations allow for ansatz spaces of arbitrary polynomial degree and its discretizations coincide with the mentioned non-conforming FEMs for the lowest polynomial degree. The discretizations presented in this talk allow not only for a uniform implementation for arbitrary m, but they also allow for lowest-order ansatz spaces, e.g., piecewise affine polynomials for arbitrary m. Besides the a priori and a posteriori analysis, the talk presents optimal convergence rates for adaptive algorithms for the new discretizations.