Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April/May 2012 **MA2VC 2011/12 A001**

UNIVERSITY OF READING

VECTOR CALCULUS (MA3VC)

1.5 hours

Attempt ALL questions

1. (a) Prove the vector differential identity:

$$\nabla \times (f\mathbf{F}) = (\nabla f) \times \mathbf{F} + f(\nabla \times \mathbf{F})$$

It is sufficient to prove the equality for the x-component of each side.

[8 marks]

(b) Demonstrate that the above identity holds for

$$f(x, y, z) = e^{xy} \quad \text{and} \quad \mathbf{F}(x, y, z) = -xy\hat{\mathbf{i}} + xy\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$$

[12 marks]

(c) Prove that $\mathbf{F}(x, y, z)$ from part (b) is not conservative by showing that it is impossible to find a scalar potential such that $\mathbf{F} = \nabla\phi$.

[5 marks]

2. (a) Prove Green's theorem:

$$\oint_{\partial R} F_x dx + F_y dy = \int_R \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) dA$$

for the special case where $F_y = 0$ and R is a simple domain defined by

$$c \leq x \leq d \quad \text{and} \quad g(x) \leq y \leq h(x)$$

where the two functions satisfy $g(c) = h(c)$ and $g(d) = h(d)$.
You can assume the fundamental theorem of calculus:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

[10 marks]

(b) Demonstrate that Green's theorem holds for

$$\mathbf{F}(x, y) = -xy\hat{\mathbf{i}} + xy\hat{\mathbf{j}}$$

where R is the first quadrant of the unit circle defined by

$$x \geq 0, \quad y \geq 0 \quad \text{and} \quad x^2 + y^2 \leq 1.$$

[15 marks]

[End of Question Paper]