Vector calculus **MA2VC** 2014–15 — Assignment 1

MA2VC: Part 2 students only.
Handed out: Tuesday 21st October. Due: **Thursday 30th October, 12 noon**.
You can use formulas and identities from the lecture notes. Do not use red pen or pencil.
Marking will be anonymous, so please write your name only on the "assessed work coversheet" and not on your work. Write your student number both on the back of the coversheet and each page of your work.
Total marks: 20. (10% of the total marks for the module.)

(1) (7 marks) Compute a scalar potential φ for the vector field

$$\vec{\mathbf{F}} = yz(z\hat{\boldsymbol{\jmath}} + y\hat{\boldsymbol{k}}).$$

Is $\vec{\mathbf{F}}$ solenoidal, irrotational? Does $\vec{\mathbf{F}}$ allow a vector potential?

(2) (6 marks) Let $\vec{\mathbf{F}}$ be a vector field with scalar potential φ, and let $\vec{\mathbf{G}}$ be a vector field with scalar potential ψ. Prove the following identity:

$$2\vec{\mathbf{F}} \cdot \vec{\mathbf{G}} = \Delta(\varphi\psi) - \psi\vec{\nabla} \cdot \vec{\mathbf{F}} - \varphi\vec{\nabla} \cdot \vec{\mathbf{G}}$$

Hint 1: You can either use some identities of those in Section 1.4 of the notes or expand the expression in components and partial derivatives. The first option is definitely simpler!
Hint 2: Identify which one of the terms in the identity can be expanded using a product rule.

(3) (7 marks) Demonstrate the identity in Exercise (2) for the vector field $\vec{\mathbf{F}}$ in Exercise (1) and the scalar field $\psi = y^3$.
Hint: The identity involves also the fields φ and $\vec{\mathbf{G}}$, you need to compute them starting from $\vec{\mathbf{F}}$ and ψ (actually you should have already computed φ).