Vector calculus MA2VC 2015–16: Assignment 1

MA2VC: Part 2 students only.
Handed out: Tuesday 20th October. Due: **Thursday 29th October, 12 noon**.
You can use formulas and identities from the lecture notes. Do not use red pen nor pencil.
Marking will be anonymous, so please write your name only on the "assessed work coversheet" and not on your work. Write your student number both on the back of the coversheet and each page of your work.
Total marks: 25. (10% of the total marks for the module.)

(Exercise 1) (8 marks) Consider the vector field $\vec{\mathbf{F}} = -x^3y^4\hat{\boldsymbol{\imath}} + 3x^2y^4z\hat{\boldsymbol{k}}$.
Compute the divergence and the curl of $\vec{\mathbf{F}}$.
Is $\vec{\mathbf{F}}$ solenoidal, irrotational?
Is $\vec{\mathbf{F}}$ conservative? If the answer is positive compute a scalar potential.
Does $\vec{\mathbf{F}}$ admit a vector potential $\vec{\mathbf{A}}$? If the answer is positive compute a potential. (In this case, look for the simplest one!)

(Exercise 2) (7 marks) Let f be a smooth scalar field. Prove the following identity:

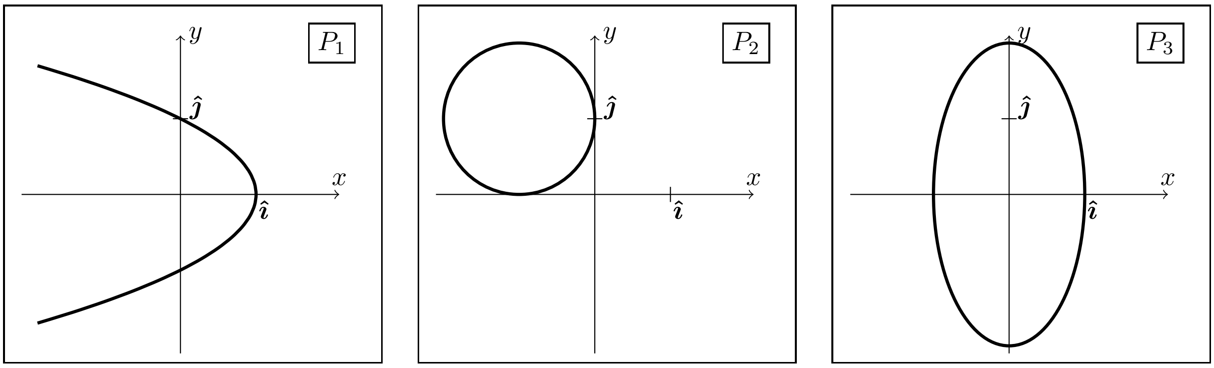
$$\vec{\nabla} \cdot \big(\vec{\nabla} f \times (\vec{\mathbf{r}} f)\big) = 0.$$

Hint: use the identities of Section 1.4 and the values of the curl and the divergence of the position vector $\vec{\mathbf{r}}$. Recall also Exercise 1.15.

(Exercise 3) (4 marks) Demonstrate the identity in Exercise 2 for the field $f = \mathrm{e}^{xy}$.

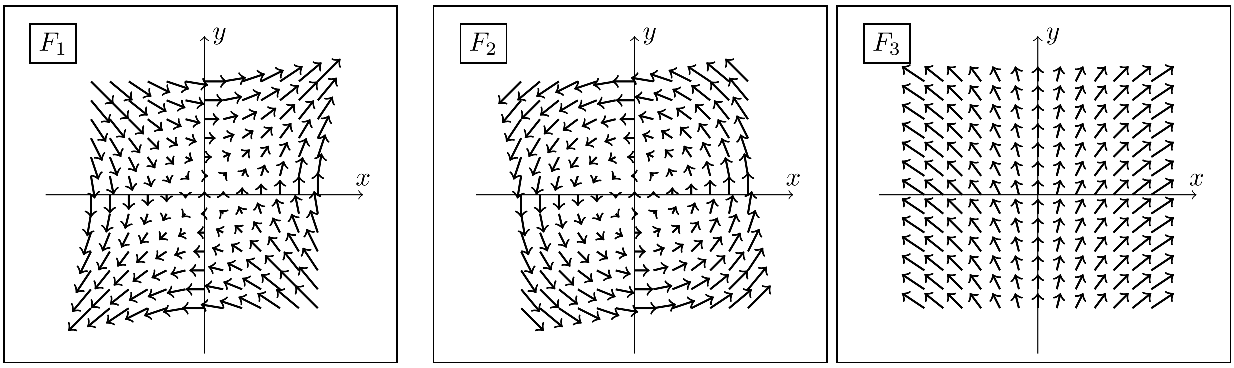
Turn the page for Exercise 4.

(Exercise 4 MA2VC) (6 marks) Consider the following three plots, representing the paths of three curves. Three of the seven curves $\vec{\mathbf{a}}_A, \dots, \vec{\mathbf{a}}_G$ listed below are parametrisations of the paths in P_1, P_2 and P_3: figure out which curve corresponds to each plot.
You do not need to justify your answer.



$$\vec{\mathbf{a}}_A(t) = (t^2+1)\hat{\boldsymbol{\imath}} + (t^4-1)\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_B(t) = (\cos t+1)\hat{\boldsymbol{\imath}} + (\sin t-1)\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_C(t) = (\cos t-1)\hat{\boldsymbol{\imath}} + (\sin t+1)\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_D(t) = \sin t\hat{\boldsymbol{\imath}} + 2\cos t\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_E(t) = t\hat{\boldsymbol{\imath}} + (t^2+1)\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_F(t) = \cos t\hat{\boldsymbol{\imath}} + \sin 2t\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{a}}_G(t) = (1-t^2)\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}}.$$

Now consider the following plots representing three planar vector fields. Which of the fields $\vec{\mathbf{G}}_A, \dots, \vec{\mathbf{G}}_G$ below correspond to figures F_1, F_2 and F_3?
Hint: look at the *signs* of the components of the fields. Recall figures 7, 11, 12, 13 in the notes.
(Note that the fields are drawn in scale: the length of the arrows is one fifth of the magnitude of the corresponding value of the field.)



$$\vec{\mathbf{G}}_A = x\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{G}}_B = \vec{\mathbf{r}},$$
$$\vec{\mathbf{G}}_C = y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{G}}_D = -\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{G}}_E = y\hat{\boldsymbol{\imath}} - x\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{G}}_F = -y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}},$$
$$\vec{\mathbf{G}}_G = -x\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}}.$$