Vector calculus MA2VC 2016–17: Assignment 1

MA2VC: Part 2 students only.
Handed out: Thursday 20th October. Due: **Thursday 27th October, 12 noon**.
You can use formulas and identities from the lecture notes. Do not use red pen nor pencil.
Marking will be anonymous, so please write your name only on the "assessed work coversheet" and not on your work. Write your student number both on the back of the coversheet and each page of your work.
Total marks: 20. (10% of the total marks for the module.)

(Exercise 1 — 7 marks)
Prove that if f is a (smooth) scalar field and $\vec{\mathbf{G}}$ is an irrotational vector field, then

$$(\vec{\nabla} f \times \vec{\mathbf{G}})f$$

is solenoidal.

Hint: Do NOT expand in coordinates and partial derivatives. Use instead the vector differential identities of §1.4 and the properties of the vector product from §1.1.2 (recall in particular Exercise 1.15).

(Exercise 2 — 5 marks)
Demonstrate that the field $(\vec{\nabla} f \times \vec{\mathbf{G}})f$ is indeed solenoidal for $f = xyz$ and $\vec{\mathbf{G}} = \vec{\mathbf{r}}$.

(Exercise 3 — 8 marks)
We define the following subsets of the xy-plane (recall that for $\vec{\mathbf{r}} \in \mathbb{R}^2$ we write $\vec{\mathbf{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$)

$$S_1 = \{\vec{\mathbf{r}} \in \mathbb{R}^2, x + y = 1\}, \qquad S_2 = \{\vec{\mathbf{r}} \in \mathbb{R}^2, x^2 + y^2 = 4\}, \qquad S_3 = \{\vec{\mathbf{r}} \in \mathbb{R}^2, x^2 = y + 1\},$$

the two-dimensional scalar fields

$$f_A(\vec{\mathbf{r}}) = \mathrm{e}^x \mathrm{e}^y, \qquad f_B(\vec{\mathbf{r}}) = x^2 - \vec{\mathbf{r}} \cdot \hat{\boldsymbol{\jmath}}, \qquad f_C(\vec{\mathbf{r}}) = \sin x \cos y + \cos x \sin y, \qquad f_D(\vec{\mathbf{r}}) = \log(|\vec{\mathbf{r}}| + \pi),$$

and the planar curves, defined for all $t \in \mathbb{R}$,

$$\vec{\mathbf{a}}_I(t) = 2\sin t\hat{\boldsymbol{\imath}} - 2\cos t\hat{\boldsymbol{\jmath}}, \quad \vec{\mathbf{a}}_{II}(t) = \cos t\hat{\boldsymbol{\imath}} + (1-\cos t)\hat{\boldsymbol{\jmath}}, \quad \vec{\mathbf{a}}_{III}(t) = (t-1)\hat{\boldsymbol{\imath}} + (t^2 - 2t)\hat{\boldsymbol{\jmath}}, \quad \vec{\mathbf{a}}_{IV}(t) = \hat{\boldsymbol{\imath}} + t^3(\hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{\imath}}).$$

- Each set S_1, S_2, S_3 is a level set of one of the fields f_A, f_B, f_C, f_D and path of one of the curves $\vec{\mathbf{a}}_I, \vec{\mathbf{a}}_{II}, \vec{\mathbf{a}}_{III}, \vec{\mathbf{a}}_{IV}$: match them.
- One of the level sets of the remaining field contains as a subset one of the three sets S_1, S_2, S_3: which one?
- The path of the remaining curve is a subset of one of the three sets S_1, S_2, S_3: which one?

You do NOT have to justify your answer.
Hint: Recall §1.2.3 and try to sketch the sets.

Please check carefully the list of common errors on page 110 of the notes and try not to commit them!
Recall also that the nabla symbol "$\vec{\nabla}$" is not a vector and cannot be treated as such.