Vector calculus MA3VC 2015–16: Assignment 1

MA3VC: Part 3 students only.
Handed out: Tuesday 20th October. Due: **Thursday 29th October, 12 noon**.
You can use formulas and identities from the lecture notes. Do not use red pen nor pencil.
Marking will be anonymous, so please write your name only on the "assessed work coversheet" and not on your work. Write your student number both on the back of the coversheet and each page of your work.
Total marks: 25. (10% of the total marks for the module.)

(Exercise 1) (8 marks) Consider the vector field $\vec{\mathbf{F}} = -x^3y^4\hat{\boldsymbol{\imath}} + 3x^2y^4z\hat{\boldsymbol{k}}$.
Compute the divergence and the curl of $\vec{\mathbf{F}}$.
Is $\vec{\mathbf{F}}$ solenoidal, irrotational?
Is $\vec{\mathbf{F}}$ conservative? If the answer is positive compute a scalar potential.
Does $\vec{\mathbf{F}}$ admit a vector potential $\vec{\mathbf{A}}$? If the answer is positive compute a potential. (In this case, look for the simplest one!)

(Exercise 2) (7 marks) Let f be a smooth scalar field. Prove the following identity:

$$\vec{\nabla} \cdot \big(\vec{\nabla} f \times (\vec{\mathbf{r}} f)\big) = 0.$$

Hint: use the identities of Section 1.4 and the values of the curl and the divergence of the position vector $\vec{\mathbf{r}}$. Recall also Exercise 1.15.

(Exercise 3) (4 marks) Demonstrate the identity in Exercise 2 for the field $f = \sin(xy + z)$.

(Exercise 4) (6 marks) Define the planar vector field $\vec{\mathbf{F}} = -2x\mathrm{e}^{-x^2}\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}}$. Compute a planar curve $\vec{\mathbf{a}}(t) = a_1(t)\hat{\boldsymbol{\imath}} + a_2(t)\hat{\boldsymbol{\jmath}}$, with $t \in \mathbb{R}$, that is perpendicular to $\vec{\mathbf{F}}$ at each point.

Hint 1: Recall that we have seen in Section 1.3.2 that some fields are orthogonal to some paths. Can you use this to compute the path of $\vec{\mathbf{a}}$? Note that $\vec{\mathbf{F}}$ is conservative!
Hint 2: Once you have the path of $\vec{\mathbf{a}}$, to find the parametrisation $\vec{\mathbf{a}}$ itself recall Remark 1.24 in the notes.