MA2VC, Vector Calculus, Assignment 2

due: 12pm, 16 Nov 2012 (late assignments will not be accepted, and marks will be deducted for poor presentation)

Consider the vector field $\mathbf{F}(\mathbf{r}) = (1-x^2)y\hat{\mathbf{j}}$.

1a) (3 marks) Calculate the line integral, $\int \mathbf{F}\cdot d\mathbf{r}$, along the straight line from $(x, y, z) = (1, 0, 0)$ to $(0, 1, 0)$.

1b) (4 marks) Calculate the line integral, $\int \mathbf{F}\cdot d\mathbf{r}$, along the circular path, $x^2 + y^2 = 1$ for $x, y \geq 0$, from $(x, y, z) = (1, 0, 0)$ to $(0, 1, 0)$.

1c) (3 marks) Show that $\mathbf{F}(\mathbf{r})$ is not conservative by evaluating $\nabla \times \mathbf{F}$.

Consider the vector field $\mathbf{F}(\mathbf{r}) = z\hat{\mathbf{i}} + 2y\hat{\mathbf{j}} + x\hat{\mathbf{k}}$.

2a) (3 marks) Calculate the line integral, $\int \mathbf{F}\cdot d\mathbf{r}$, along the straight line from $(x, y, z) = (0, 0, 0)$ to $(1, 1, 1)$.

2b) (3 marks) Calculate the line integral, $\int \mathbf{F}\cdot d\mathbf{r}$, along the intersection of $y = x^2$ and $z = x^3$ from $(x, y, z) = (0, 0, 0)$ to $(1, 1, 1)$.

2c) (4 marks) Show that $\mathbf{F}(\mathbf{r})$ is conservative by finding a scalar potential, $\phi(\mathbf{r})$. Then use the potential to evaluate the line integral, $\int \mathbf{F}\cdot d\mathbf{r}$, from $(x, y, z) = (0, 0, 0)$ to $(1, 1, 1)$.