Vector calculus MA2VC 2013–14 — Assignment 1

Handed out: Friday 18th October. Due: **Friday 1st November, 12 noon**.
Late assignments will not be accepted, marks will be deducted for poor presentation.
You can use the vector identities in the lecture notes.

(1) (5 marks) Prove the following identity:

$$\vec{\nabla} \cdot \big(\vec{\mathbf{r}} f g\big) = 3fg + (\vec{\mathbf{r}} \cdot \vec{\nabla} f)g + (\vec{\mathbf{r}} \cdot \vec{\nabla} g)f,$$

where $\vec{\mathbf{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$ is the position vector, and $f(\vec{\mathbf{r}})$ and $g(\vec{\mathbf{r}})$ are two scalar fields.
Hint: you can either use the vector differential identities in the boxes of Propositions 1.35 and 1.36 in the notes, or the definitions of gradient and divergence.

(2) (5 marks) Demonstrate the above identity for the scalar fields

$$f = e^{xy}, \qquad g = y^4 z.$$

(3) (5 marks) Verify that the vector field

$$\vec{\mathbf{F}} = y^2\hat{\boldsymbol{\imath}} + z^3\hat{\boldsymbol{\jmath}}$$

is solenoidal and find a vector potential $\vec{\mathbf{A}}$.
Hint: look for a potential such that two of its three components vanish.

(4) (5 marks) Show that, for any unit vector $\hat{\boldsymbol{n}}$, and any vector $\vec{\mathbf{w}}$ perpendicular to $\hat{\boldsymbol{n}}$, the identity

$$\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \vec{\mathbf{w}}) = -\vec{\mathbf{w}}$$

holds true. You can make use of the identities in Section 1.1.2 of the notes. Demonstrate this identity for the vectors

$$\hat{\boldsymbol{n}} = \frac{3}{5}\hat{\boldsymbol{\imath}} + \frac{4}{5}\hat{\boldsymbol{k}}, \qquad \vec{\mathbf{w}} = 3\hat{\boldsymbol{\jmath}}.$$