Vector calculus — Final results of previous assignments (2011–15)

Note that for some exercises other results are possible, e.g. vector potentials.

Assignment	Question	Final result
2011 assignment 1	1	proof
	2	$y^2 - x^2$
	3a	$y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}}$
	3b	proof
	3c	$\phi = xy + \lambda$
2011 assignment 2	1a	$4t^3 \mathrm{e}^{t^4}$
	1b	$4\mathrm{e}(0.01)$
	2a	$-\vec{\mathbf{r}}/\lvert\vec{\mathbf{r}}\rvert^3$
	2b–d	$-1/2\sqrt{2}$
2011 assignment 3	1a–b	$27/8$
	2a–b	$1/28$
2011 assignment 4	1	27
2012 assignment 1	1	tetrahedron, $-\hat{\boldsymbol{\imath}}, -\hat{\boldsymbol{\jmath}}, -\hat{\boldsymbol{k}}, (3\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}})/\sqrt{14}$
	2	proof
	3	$(x-y)\mathrm{e}^{x+y+z}$
	4	$\phi = x^2 z + y^2 z + \frac{1}{3}z^3 + \lambda$
	5	e.g. $\vec{\mathbf{A}} = xy^2\hat{\boldsymbol{k}}$
2012 assignment 2	1a	$5/12$
	1b	$1/4$
	1c	$\vec{\nabla} \times \vec{\mathbf{F}} = -2xy\hat{\boldsymbol{k}}$
	2a–b	2
	2c	$\phi = xz + y^2 + \lambda$
2012 assignment 3	1	3
	2a–b	$-81\pi/4$
2012 assignment 4	1a	3
	1b	proof
	2a–b	$\pi/4$
2013 assignment 1	1	proof
	2	$8y^4 z e^x y + 2xy^5 z e^{xy}$
	3	e.g. $\vec{\mathbf{A}} = (\frac{1}{3}y^3 - xz^3)\hat{\boldsymbol{k}}$
	4	proof
2013 assignment 2	1	$\frac{d\vec{\mathbf{a}}}{dt} = \mathrm{e}^t\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + \mathrm{e}^{2t}(2\cos t - \sin t)\hat{\boldsymbol{k}}$, $\frac{d(f(\vec{\mathbf{a}}))}{dt}(t) = \frac{e^{2t}+t+e^{4t}(2\cos^2 t - \cos t \sin t)}{\sqrt{e^{2t}+t^2+e^{4t}\cos^2 t}}$
	2	$5\pi\sqrt{2}$
	3	$\int_{\Gamma_c} \vec{\mathbf{G}} \cdot \mathrm{d}\vec{\mathbf{r}} = 4/\pi, \quad \int_{\Gamma_d} \vec{\mathbf{G}} \cdot \mathrm{d}\vec{\mathbf{r}} = 4/3$
	4	$\varphi = \frac{1}{2}x^{+}\frac{1}{3}y^3 + \frac{1}{4}z^4 + \lambda, \quad \int_{\Gamma_c} \vec{\mathbf{H}} \cdot \mathrm{d}\vec{\mathbf{r}} = 0$
2013 assignment 3	1a	$\xi = xy,\ \eta = y$ (see exercise in §2 of lecture notes)
	1b	$1/4$
	2	$\pi/6$
	3	3
	4	Each expression contains a mistake, find it!
2013 assignment 4	1	2
	2	124π
	3	-1/3

2014 assignment 1 MA2VC	1	$\varphi = \frac{1}{2}y^2z^2 + \lambda$, irrotational, non solenoidal, no v.p.
	2	proof
	3	$6y^3z^2$
2014 assignment 1 MA3VC	1	$\varphi = \frac{1}{2\pi}\sin(2\pi x) + \frac{1}{2}y^2z^2 + \lambda$, irrotational, non sol., no v.p.
	2	proof
	3	$6y^3z^2$
	4	$\hat{\boldsymbol{n}} = (y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}})/\sqrt{y^2+z^2}$
2014 assignment 2 MA2VC	1	2
	2	A→2, B→32, C→0, D→-2, proof
	3	4
	4	proof
2014 assignment 2 MA3VC	1–3	same as MA2VC
	4	$\vec{\mathbf{a}} = \cos t\hat{\boldsymbol{\imath}} + \sin t\hat{\boldsymbol{\jmath}} + (\cos^2 t - \sin^2 t)\hat{\boldsymbol{k}}$,
		$\hat{\boldsymbol{\tau}} = \frac{-\sin t\hat{\boldsymbol{\imath}} + \cos t\hat{\boldsymbol{\jmath}} - 4\cos t\sin t\hat{\boldsymbol{k}}}{\sqrt{\sin^2 + \cos^2 t + 16\sin^2 t\cos^2 t}}$, compatible with $\hat{\boldsymbol{n}}$
2015 assignment 1 MA2VC	1	$\vec{\nabla}\cdot\vec{\mathbf{F}} = 0$, $\vec{\nabla}\times\vec{\mathbf{F}} = 12x^2y^3z\hat{\boldsymbol{\imath}} - 6xy^4z\hat{\boldsymbol{\jmath}} - 4x^3y^3\hat{\boldsymbol{k}}$, solenoidal,
		not irrotational, not conservative, no s.p., v.p.: $\vec{\mathbf{A}} = x^3y^4z\hat{\boldsymbol{\jmath}}$
	2	proof
	3	demonstration
	4	$P_1 : \vec{\mathbf{a}}_G$, $P_2 : \vec{\mathbf{a}}_C$, $P_3 : \vec{\mathbf{a}}_D$, $F_1 : \vec{\mathbf{G}}_C$, $F_2 : \vec{\mathbf{G}}_F$, $F_3 : \vec{\mathbf{G}}_A$.
2015 assignment 1 MA3VC	1–2	same as MA2VC
	3	demonstration
	4	$\vec{\mathbf{a}}(t) = t\hat{\boldsymbol{\imath}} + (\mathrm{e}^{-t^2} + \lambda)\hat{\boldsymbol{\jmath}}$.
2015 assignment 2 MA2VC	1	Area = 4/3, figure P_3
	2.1	1/2
	2.2	1/2
	2.3	proof
	2.4	$\vec{\mathbf{a}}(t) = (1-t)\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}} + t(1-t)\hat{\boldsymbol{k}}$
	3	F, T, T, F, T
2015 assignment 2 MA3VC	1–2	same as MA2VC
	3	same as MA2VC, plus justification