## Vector calculus 2016-17 - Solutions to previous exams

These are the results of the questions appeared in previous vector calculus exams, which are available on the course web page. They are provided to double check the solutions you found in your revision. Note that these are only the final values, in a real exam you obtain almost all the points for the solution procedure, not for the final results! Where you read "proof", the corresponding question required a proof which is not reported here. For the questions of the type "demonstrate the identity...", we give here the results you should get for both the left- and the right-hand side of the identity. Note that the notation used in previous years might be slightly different from that you are used to; for example, in older exams double and triple integrals were denoted with a single integral sign.

| Exam | Question | Final result |
| :---: | :---: | :---: |
| MA2VC 2011-12 | $\begin{aligned} & 1 \mathrm{a} \\ & 1 \mathrm{~b} \\ & 2 \mathrm{a} \\ & 2 \mathrm{~b} \end{aligned}$ | proof $\begin{aligned} & x z^{2} \mathrm{e}^{x y} \hat{\boldsymbol{\imath}}-y z^{2} \mathrm{e}^{x y} \hat{\boldsymbol{\jmath}}+\left(x+y+x y^{2}+y x^{2}\right) \mathrm{e}^{x y} \hat{\boldsymbol{k}} \\ & \text { proof } \\ & 1 / 3 \end{aligned}$ |
| MA3VC 2011-12 | $\begin{aligned} & 1 \mathrm{a} \\ & 1 \mathrm{~b} \\ & 1 \mathrm{c} \\ & 2 \mathrm{a} \\ & 2 \mathrm{~b} \\ & \hline \end{aligned}$ | proof $x z^{2} \mathrm{e}^{x y} \hat{\boldsymbol{\imath}}-y z^{2} \mathrm{e}^{x y} \hat{\boldsymbol{\jmath}}+\left(x+y+x y^{2}+y x^{2}\right) \mathrm{e}^{x y} \hat{\boldsymbol{k}}$ proof proof $2 / 3$ |
| MA2VC 2012-13 | $\begin{array}{r} 1 \mathrm{a} \\ 1 \mathrm{~b} \\ 2 \mathrm{a} \\ 2 \mathrm{~b} \\ 3 \\ 4 \end{array}$ | proof $x \hat{\boldsymbol{\imath}}-y \hat{\boldsymbol{\jmath}}+(y-x) \hat{\boldsymbol{k}}$ <br> 7 $\phi=x y z+c$ <br> proof $\frac{1}{2}(\hat{\boldsymbol{\imath}}+\hat{\boldsymbol{\jmath}})$ |
| MA3VC 2012-13 | $\begin{array}{r} \hline 1 \mathrm{a} \\ 1 \mathrm{~b} \\ 2 \\ 3 \mathrm{a} \\ 3 \mathrm{~b} \\ 4 \end{array}$ | proof $-2 \hat{\boldsymbol{\imath}}-2 \hat{\boldsymbol{k}}$ <br> $\pi / 2$ <br> proof <br> proof $\frac{1}{2}(\hat{\boldsymbol{\imath}}-\hat{\boldsymbol{\jmath}})$ |
| MA2VC 2013-14 | $\begin{array}{r} \hline 1 \mathrm{a} \\ 1 \mathrm{~b} \\ 1 \mathrm{c} \\ 2 \\ 3 \\ 4 \end{array}$ | proof first term is $-4 x \cos 2 z \hat{\boldsymbol{\jmath}}$ proof $7 / 2$ $5 / 3$ proof |
| MA3VC 2013-14 | $\begin{array}{r} \hline 1 \mathrm{a} \\ 1 \mathrm{~b} \\ 1 \mathrm{c} \\ 2 \\ 3 \mathrm{a} \\ 3 \mathrm{~b} \\ 4 \\ \hline \end{array}$ | proof <br> first term is $6 x^{2} y^{2} \cos 2 z \hat{\boldsymbol{\imath}}-4 x y^{3} \cos 2 z \hat{\boldsymbol{\jmath}}-3 x^{2} y^{2} \sin x \hat{\boldsymbol{k}}$ proof <br> $7 / 2$ <br> $5 / 3$ <br> proof <br> proof |
| MA2VC 2014-15 | 1 a 1 b 1 c 2 3 4 5 5 | $\begin{aligned} & \text { proof } \\ & 2 x y^{2} \mathrm{e}^{x} \\ & \text { proof } \\ & \sqrt{2} \\ & 5 \log ^{2} 2 \\ & 4 \pi \\ & \text { proof } \\ & \text { proof } \\ & \hline \end{aligned}$ |
| MA3VC 2014-15 | 1 a 1 b 1 c 2 3 4 5 6 | ```proof \(2 \mathrm{e}^{x y+z}\left(x^{2}-y^{2}\right)(z-x y)\) proof, \(\overrightarrow{\mathbf{A}}=-\frac{1}{2} \psi \vec{\nabla}\left(\varphi^{2}\right)\) \(2 \sqrt{2}\) \(5 \log ^{2} 2\) \(a^{2} \pi\) proof proof``` |


| Exam | Question | Final result |
| :---: | :---: | :---: |
| MA2VC 2015-16 | 1 a | $\overrightarrow{\mathbf{a}}(t)=\cos t \hat{\imath}+\sin t \hat{\boldsymbol{\jmath}},-\pi / 2<t<\pi / 2$ |
|  | 1b | $2 / 3$ |
|  | 1 c | $\pi / 2$ |
|  | 2 a | proof |
|  | 2b | 9/20 |
|  | 2c | proof |
|  | 3 | proof |
|  | 4 | $1 / 2$ |
|  | 5 | $\pi / 6$ |
|  | 6 | proof |
| MA3VC 2015-16 | 1 a | $\overrightarrow{\mathbf{a}}(t)=\cos t \hat{\imath}+\sin t \hat{\boldsymbol{\jmath}},-\pi / 2<t<\pi / 2$ |
|  | 1b | 0 |
|  | 1c | 4/3 |
|  | 2 a | proof |
|  | 2 b | 11/40 |
|  | 2 c | proof |
|  | 3 | proof |
|  | 4 | $1 / 2$ |
|  | 5 | $\pi a^{3} / 6$ |
|  | 6 | proof |

