Master Program in Electronic Engineering

Advanced Mathematical Methods for Engineers

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1. Let $y_0 \in \mathbf{R}$ and consider the following Cauchy Problem

$$\begin{cases} y'(x) = \dfrac{y(x)}{1+y^2(x)} \\ y(0) = y_0. \end{cases}$$

a) Discuss local and global existence and uniqueness of solutions.

b) Draw a qualitative graph of solutions establishing, in particular, if there are orizontal asymptots.

2. Consider for $\alpha \neq 1$ the following nonlinear ODE system

$$\begin{cases} x' = -x + \mathrm{e}^{\alpha y} - 1 \\ y' = \mathrm{e}^{x-y} - 1. \end{cases}$$

a) Verify that $(0,0)$ is a critical point of the system.

b) Discuss its asymptotic stability depending on the values of $\alpha \neq 1$.

3. Consider in $[0,1]$ the sequences of functions

$$f_n(x) = \frac{x^n}{1+x^{10}}$$

and

$$g_n(x) = \frac{x^{1/n}}{1+x^2}.$$

a) Compute the limit f *a.e.* of f_n.

b) Compute the limit g *a.e.* of g_n.

c) Establish if it is true or not that $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 f(x)\,dx$ justifying it.

d) Establish if it is true or not that $\lim_{n\to\infty} \int_0^1 g_n(x)\,dx = \int_0^1 g(x)\,dx$ justifying it.

4. Prove that for $x \in \mathbf{R}$ and $\tau \in \mathbf{R}^+$ we have as $\tau \to 0$ the following two convergences in $\mathcal{D}'$:

a) $\frac{1}{2}\log(\tau^2 + x^2) + \arctan\left(\frac{x}{\tau}\right) \to \log(|x|) + \frac{\pi}{2}\mathrm{sign}(x)$ and

b) $\frac{x+\tau}{x^2+\tau^2} \to \mathrm{pv}(\frac{1}{x}) + \pi\delta.$