Master Program in Electronic Engineering

# Advanced Mathematical Methods for Engineers

## June 26, 2018

**1.** Given the ODE

$$y''(t) - 4y'(t) + 5y(t) = e^{2t}(1 + \cos t) + 5t^2 \tag{1}$$

1.1) Find all solutions of the corresponding homogeneous equation $y''(t) - 4y'(t) + 5y(t) = 0$.

1.2) Find one particular solution of the ODE (1).

**2.** Given, for $\alpha \in \mathbf{R}$, the ODE system

$$\begin{cases} x' = x - (\alpha^2 + 2)y \\ y' = x + \alpha y \end{cases}$$

2.1) Find the values of $\alpha$ such that all solutions are bounded on $[0, +\infty)$.

2.2) Find the values of $\alpha$ such that such there are solutions (not identially equal to 0) bounded on the whole $\mathbf{R}$.

**3.** Consider in $(0, +\infty)$ the sequence of functions

$$f_n(x) = \frac{1}{2 + x^n}$$

and prove that

3.1) $f_n \in L^1(0, +\infty)$ for every $n \geq 2$,

3.2) Find $f$ such that $f_n \to f$ as $n \to \infty$ pointwise in $(0, +\infty)$,

3.3) Compute (justifying the computations) the $\lim_{n\to+\infty} \int_0^\infty f_n(x)dx$.

**4.** Let $f$ be a distribution in $\mathbf{R}$: $f \in \mathcal{D}'(\mathbf{R})$. Prove the two following propositions.

4.1) $f$ is an even distribution if and only if $f'$ is odd.

4.2) If $f$ is an even distribution, then it is null in correspondence of any test funtion $v \in \mathcal{D}(\mathbf{R})$ odd.