

Master Program in Electronic Engineering
Advanced Mathematical Methods for Engineers

June 28, 2022

1. Consider the following Cauchy Problem

$$\begin{cases} y'(x) = -y(x)x^2 + 4x^2 \\ y(0) = 5. \end{cases}$$

- a) Discuss local and global existence and uniqueness of solutions.
- b) Draw the graph of the solution.
- c) Compute the value $S = 4(y'(1) + y(1))$.

2. Given the nonlinear ODE system

$$\begin{cases} x'(t) = x(t)(y(t) - 1) \\ y'(t) = y(t)(2x(t) + y(t) - 5) \end{cases}$$

find the stationary points and discuss their stability.

3. Let $f_n : [0, +\infty) \rightarrow \mathbf{R}$ defined as

$$f_n(x) = n^2 x^{1/2} e^{-nx}.$$

- a) Compute the norm $\|f_n\|_\infty := \sup\{|f_n(x)| : x \in [0, +\infty)\}$.
- b) Compute the pointwise limit f of f_n on $[0, +\infty)$ as $n \rightarrow \infty$.
- c) Is it true that $f_n \rightarrow f$ uniformly on $[0, +\infty)$? (justify the answer)
- d) Is it true that $f_n \rightarrow f$ uniformly on $[1, +\infty)$? (justify the answer)

4. Compute the Fourier transforms of

- a) e^{ix} ,
- b) $\sin(x)$ (use step a)),
- c) $\cos(x)$ (use step a)),
- d) $\text{sign}(x)$

justifying the answers with computations.