Parameter identification for a nonlinear parabolic boundary value problem with a blowing up diffusivity

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Abstract

The paper deals with an identification problem arisen from the theory of water infiltration in soils. The scope is to determine the rain rate that produces the water infiltration into an unsaturated soil in which saturation can be possibly reached after sometime. The rain history is identified from available recorded moisture observations in the soil and we focus here on situations with very scarce data, up to a unique recording made in the flow domain at the final time.

The infiltration mathematical model consists in a parabolic boundary value problem with a blowing up diffusion coefficient that models the simultaneous saturated-unsaturated flow and the identification problem will be solved as a control problem, (P).

The existence, uniqueness and regularity results for the solution of the state system are proved in an appropriate functional space. After proving the existence of a solution to problem (P) we shall introduce a family of approximating identification problems (P_ε) which approach problem (P) in some sense and whose solutions converge to a solution of problem (P).

The optimality conditions will be determined first for the approximating problem and using the adapted penalization method will be deduced for the original problem too. Some mathematical difficulties, induced by the particularities of the free boundary problem represented by the state system, will arise in the 3-D case so that in this case the solution of the dual system will be obtained as a weak limit of the solutions to the approximating dual system. However, for a unsaturated initial data, the equations describing the dual system can be derived in the 1-D case.