STATIONARY PROBLEM FOR THE COMPLEX GINZBURG-LANDAU EQUATION

Noboru Okazawa
Science University of Tokyo

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Abstract

In this talk we are concerned with the initial-Dirichlet boundary value problems for the **complex Ginzburg-Landau equation**:

$$\text{(CGL)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^{q-2}u - \gamma u = 0, & (x,t) \in \Omega \times \mathbb{R}_+, \\ u(0,t) = 0, \quad t \in \mathbb{R}_+, \\ u(x,0) = u_0(x), \quad x \in \Omega. \end{cases}$$

The equation is recognized with those complex coefficients in front of the Laplacian and nonlinear term. In particular, if

$$0 \le \frac{|\beta|}{\kappa} \le c_q := \frac{2\sqrt{q-1}}{q-2}, \tag{1}$$

then the L^2-strong wellposedness has been established satisfactorily. In fact, the mapping $u \mapsto -(\lambda+i\alpha)\Delta u + (\kappa+i\beta)|u|^{q-2}u$ is m-accretive (maximal monotone) in $L^2(\Omega)$. Moreover, subdifferential operator approach yield stronger results including smoothing effect.

Last year I reported that even when condition (1) breaks down, i.e.,

$$c_q < \frac{|\beta|}{\kappa} < +\infty, \tag{2}$$

the solution operators form a nonlinear non-contraction semigroup on $L^2(\Omega)$ provided that a strong restriction is imposed on the power of nonlinearity:

$$2 \le q \le 2 + \frac{4}{N}. \tag{3}$$

However, any kind of **generation theory** is not yet known for the non-contraction semigroup associated with (CGL) under conditions (2) and (3).

The purpose of this talk is to report some preliminary consideration for the estimate of iterations of the solution operators ("resolvents") to the **stationary problem**:

$$\text{(StCGL)} \qquad \begin{cases} \zeta u - (\lambda + i\alpha)\Delta u + (\kappa + i\beta)|u|^{q-2}u - \gamma u = 0, & x \in \Omega, \\ u(x) = 0, \quad x \in \partial\Omega \end{cases}$$

associated with (CGL).