The controllability of the Gurtin-Pipkin equation

L. Pandolfi, Politecnico di Torino

The Gurtin Pipkin equation is the equation

$$\theta_t(t,x) = \int_0^t b(t-s)\Delta\theta(s,x)\,ds \qquad x \in \Omega\,,\ t \geq 0\,. \tag{1}$$

Here Δ denotes the laplacian in the variable x. This equation has been proposed in order to keep into account memory effects in heat transfer and it turns out that it displays an hyperbolic behavior.

We assume that Eq. (1) is subject to a Dirichlet control,

$$\theta_{|\partial\Omega} = u \tag{2}$$

and we assume the initial state to be zero. We prove that the control process described by (1)-(2) is exactly controllable process: *there exists T_0 such that for every $\theta_0 \in L^2(\Omega)$ we can find an input u such that $\theta(T,x) = \theta_0(x)$.*

The proof uses an idea proposed by Belleni (in the case of homogeneous boundary conditions and in the presence of an affine term): it is possible to use the cosine operator theory in order to reduce the solution of the Gurtin-Pipkin equation to the solution of a Volterra integral equation whose dominant term is the input to state map of

$$w_{tt} = \Delta w \quad x \in \Omega\,, \qquad w_{|\partial\Omega} = u\,.$$

It is known that for T large enough this map is surjective.

We represent the solution of the Volterra integral equation corresponding to (1)-(2) using the usual Newmann series. It turns out that, for T large enough, the solution at time T is the sum of a surjective plus a compact operator so that its image is closed with finite dimensional codimension.

The ortogonal to the reachable set is then characterized and, using the fact that the laplacian with both Dirichlet and Newmann conditions has zero solution (under weak conditions on Ω) we see that the reachable set is dense in $L^2(\Omega)$ for T large enough. Controllability follows from here.