Modeling and control of curved mechanical structures

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In this lecture, we study new mathematical models for the deformation of curved mechanical structures such as arches, curved rods, and shells, under given loads. We always aim to develop a meaningful theory under low regularity assumptions on the geometry of the mechanical structures. In particular, Naghdi-type models of polynomial type are derived for rods and shells.

For rods, the resulting elasticity system consists of a boundary value problem for a system of nine coupled ordinary differential equations, while in the case of shells there results a boundary value problem for a coupled system of six partial differential equations in two variables. Owing to the fact that we are dealing with curved structures that have to be represented by curvilinear coordinates, both systems have a rather involved structure that renders the proof of coercivity of the associated bilinear forms difficult, since Korn's inequality can no longer be applied directly.

For both rods and shells coercivity can be shown provided that the structures are "sufficiently thin". We have also been able to study optimal control problems for the resulting systems, deriving existence of optimal controls and first-order necessary optimality conditions. In the case of rods the obtained results have been verified in numerous numerical examples.

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