Carleman estimates for Lamé systems for functions without compact supports and the application to inverse problems

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For functions without compact supports, we established Carleman estimates for the two-dimensional non-stationary Lamé system with the Dirichet or the stress boundary condition:

$$\begin{aligned} &\rho(x)\frac{\partial^2 \mathbf{u}}{\partial t^2} - \mu(x)\Delta\mathbf{u} - (\mu(x)+\lambda(x))\nabla \mathrm{div}\, \mathbf{u} \\ - \;\; &(\mathrm{div}\, \mathbf{u})\nabla\lambda(x) - (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)\nabla\mu(x) = \mathbf{f} \quad \text{in } Q = (0,T)\times\Omega, \end{aligned}$$

with

$$\left(\sum_{j=1}^{2} n_j\sigma_{j1}, \sum_{j=1}^{2} n_j\sigma_{j2}\right)^T = \mathbf{g} \quad \text{on } (0,T)\times\partial\Omega$$

or

$$\mathbf{u} = \mathbf{g} \quad \text{on } (0,T)\times\partial\Omega$$

where $\mathbf{u} = (u_1, u_2)^T$, $\mathbf{f} = (f_1, f_2)^T$ are the vector functions, U^T denotes the transpose of the vector U, Ω is a bounded domain in R^2, $(n_1, n_2)^T$ is the unit outward normal vector to $\partial\Omega$ and

$$\sigma_{jk} = \lambda(x)\delta_{jk}\mathrm{div}\, \mathbf{u} + \mu(x)\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right).$$

Then we apply them to inverse problems of determining $\rho(x)$, $\lambda(x)$ and $\mu(x)$ from interior measurements with a single suitable choice of boundary value and initial value.